

MASS GENERATION IN THREE DIMENSIONS

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Abstract

I consider the dilute monopole gas expansion of the three dimensional Yang-Mills-Higgs system in the symmetry broken phase. The functional determinants which occur in such an expansion are computed in the heat kernel approximation for an arbitrary $SU(N)$ gauge group. Explicit expressions for the gauge boson mass in the unbroken gauge sector and the string tension are obtained for the $SU(2)$ gauge model and are evaluated numerically. The results show a strong dependence on the ratio m_{Higgs}/m_W .

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1 Introduction

The dynamics of three and four dimensional systems differ substantially. A characteristic difference is that the infrared modes play a more important role in the lower dimensional case. On the one hand this simplifies the perturbative investigations since the ultra-violet regime is less important and, by that, the separation of the non-physical, ultra-violet divergences becomes easier. On the other hand, however, the infrared modes may render the usual perturbation expansion divergent order by order in the infrared. In such a case we are forced to perform a partial resummation of the perturbation expansion. The resulting improved perturbation expansion is based on certain quasi-particles with screened interactions and may be very different from the original perturbation expansion.

These phenomena can nicely be studied by means of the three dimensional $SU(N)$ Yang-Mills-Higgs system in the symmetry broken phase. The action of this model is given by

$$S[A_i^a, \Phi^a] = \int d^3x \left(\frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{2} (D_i \Phi)^a (D_i \Phi)^a + \frac{\lambda}{8} (\Phi^{a2} - v^{a2})^2 \right), \quad (1)$$

where

$$F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + e f^{abc} A_i^b A_j^c, \quad (2)$$

$$(D_i \Phi)^a = \partial_i \Phi^a + e f^{abc} A_i^b \Phi^c,$$

f^{abc} are the structure constants of the $SU(N)$ gauge group and the coupling constants e^2 and λ have the dimension of an energy. Since the theory is super-renormalizable one only needs a single counterterm for the mass renormalization of the Higgs field Φ . But the perturbation expansion is order by order divergent in the infrared and thus renders the theory essentially nonperturbative.

Fortunately, the action (1) possesses stable localized extrema, the static Yang-Mills-Higgs monopole configurations of the corresponding four dimensional gauge model [1, 2, 3, 4, 5]. The polarization of a dilute gas of such pseudo-particles screens the long range magnetic Coulomb forces so that the semiclassical expansion yields an infrared stable approximation [6]. According to this expansion the

electric charges are confined by a linear potential and the gauge bosons acquire a non-vanishing mass.

Originally, the reason for studying the $SU(2)$ dilute monopole gas expansion was the fact that QED in the Georgi-Glashow model is a simple example of a compact $U(1)$ gauge theory. Polyakov's work [6] in 1976 had initiated an extensive study of the three dimensional compact $U(1)$ gauge theory and in particular of a lattice version of this $U(1)$ theory that is obtained by using the Villain approximation of the Wilson action [7]. The primary goal of these lattice studies was and still is to reveal the phase structure of such theories by studying the behavior of quantities such as the string tension as a function of the inverse temperature $\beta = 1/e^2 a$ where a is the lattice spacing.

But there are various other important aspects of the three dimensional model (1) which require a detailed investigation of the dilute monopole gas expansion in the context of the model itself. One certainly is the fact that it resembles the effective model for the high temperature phase of QCD [8, 9, 10]. Through dimensional reduction of the four dimensional gauge theory of strong interactions at finite temperature one arrives at a three dimensional gauge model in which the zero-component of the gauge field plays the role of the Higgs field. The Higgs sector is screened by the Debye mass whereas the gauge sector is infrared divergent. Therefore conventional perturbation expansion is inapplicable. To overcome this difficulty one has to perform a certain partial resummation of infinitely many Feynman diagrams. Such a resummation could be very similar to the dilute monopole gas expansion.

Another important issue is the question of the true vacuum. In most computations of the effective potential only the trivial constant background field is considered. However it is well-known since the work of Coleman [11, 12] that nontrivial saddle point configurations of the underlying theory may alter these results fundamentally in the sense that what appears to be the vacuum state is actually unstable. For instance if we consider the three dimensional Yang-Mills-Higgs model in the symmetrical phase the one loop effective potential obtained with a constant background field shows no symmetry breaking effects. This may

change if we also include non-trivial saddle point configurations in the computation. Such nontrivial saddle point configurations arise because of the explicit symmetry breaking source term that we commonly introduce in order to study spontaneous symmetry breaking. The form of these configurations is very similar to the 't Hooft-Polyakov monopoles and their contributions to the effective potential can be computed in the dilute gas expansion. However, in order to investigate such a possibility of spontaneous symmetry breaking we first must have a complete description of the dilute monopole gas.

The present paper contains further details of the dilute monopole gas expansion which was sketched in Ref. [6]. Its goal is to provide the formulae for the quantitative analysis of the expansion. This is achieved by an approximate computation of the 1-loop determinants which occur in the monopole partition function. We regularize the determinants with the help of the ζ -function regularization and then evaluate the resulting expressions in the heat kernel approximation [13, 14]. This enables us to present explicit expressions for the gauge boson mass in the unbroken $U(1)$ gauge sector and the string tension in the $SU(2)$ gauge model.

The numerical evaluation of the formulas shows that the $U(1)$ gauge boson mass and the string tension strongly depend on the ratio $\lambda/e^2 = m_{\text{Higgs}}^2/m_W^2$ where m_W is the heavy vector boson mass. In both limits, $\lambda/e^2 \rightarrow 0$ and $\lambda/e^2 \rightarrow \infty$, the $U(1)$ gauge boson mass and the string tension vanish but physics is different. We shall argue that in the Prasad-Sommerfield limit $\lambda/e^2 \rightarrow 0$ the symmetry is restored whereas the limit $\lambda/e^2 \rightarrow \infty$ corresponds to a non-confining Higgs phase with a massless $U(1)$ gauge boson.

Since there exist several misprints in the original literature and since the precise expressions which arise in the formulation of the dilute monopole gas expansion are important for our considerations we find it appropriate to present the dilute monopole gas expansion in greater detail. In Section 2 we rederive the one-monopole partition function for the $SU(N)$ Yang-Mills-Higgs theory. The evaluation of the $SU(N)$ one-loop determinants with the help of the heat kernel method is subject of Section 3. In Section 4 we give the precise expressions for

the $SU(2)$ dilute monopole gas partition function, the gauge boson mass in the unbroken gauge sector and the string tension. Numerical results for the $SU(2)$ gauge model are presented in Section 5 and a summary is given in Section 6.

2 The $SU(N)$ one-monopole partition function

The general presupposition for the semi-classical expansion is the existence of non-trivial saddle point configurations of the underlying classical action. Since the action (1) of the three dimensional $SU(N)$ Yang-Mills-Higgs system is just the energy of the static configurations of the corresponding four dimensional system, the static solitons of the four dimensional theory – the Yang-Mills-Higgs magnetic monopoles – are the saddle points or pseudo-particles of the three dimensional model.

In order to obtain the one-monopole contribution to the functional integral,

$$Z = \int \mathcal{D}[A_i^a] \mathcal{D}[\Phi^a] \exp(-S[A_i^a, \Phi^a]) , \quad (3)$$

in the one-loop approximation one writes the fields A_i^a (Φ^a) as a sum of the classical one-monopole field \hat{A}_i^a ($\hat{\Phi}^a$) and a quantum fluctuation a_i^a (ϕ^a) and expands the action up to terms quadratic in the quantum fluctuations. As usual the functional integration over the quantum fluctuations requires some gauge-fixing. A suitable choice is the background-field gauge defined by the gauge-fixing action

$$S_{\text{gf}}[A_i^a, \Phi^a] = \frac{1}{2} \int d^3x \left(\widehat{D}_i^{ab} (A_i^b - \hat{A}_i^b) + e f^{abc} \widehat{\Phi}^b (\Phi^c - \hat{\Phi}^c) \right)^2 . \quad (4)$$

The corresponding Faddeev-Popov determinant is given by

$$\det[\mathcal{M}_{\text{FP}}] = \det \left[\widehat{D}_i^{ac} D_i^{cb} + e^2 f^{acd} f^{dc'b} \widehat{\Phi}^c \Phi^{c'} \right] , \quad (5)$$

which, in a one-loop computation can be approximated by

$$\det[\mathcal{M}_{\text{FP}}] \approx \det \left[\widehat{D}_i^{ac} \widehat{D}_i^{cb} + e^2 f^{acd} f^{dc'b} \widehat{\Phi}^c \widehat{\Phi}^{c'} \right] . \quad (6)$$

If one performs the standard algebraic manipulations and employs the classical field equations

$$\begin{aligned}(\widehat{D}_i \widehat{F}_i)^a &= e f^{abc} \widehat{\Phi}^b (\widehat{D}_j \widehat{\Phi})^c, \\ (\widehat{D}_i \widehat{D}_i \widehat{\Phi})^a &= \frac{\lambda}{2} (\widehat{\Phi}^{a2} - v^{a2}) \widehat{\Phi}^a,\end{aligned}\tag{7}$$

as well as the Bianchi identity for the $SU(N)$ structure constants one arrives at the following expression for the functional integral in the one-monopole sector

$$Z_1 = \int \mathcal{D}[a_i^a] \mathcal{D}[\phi^a] \det[\mathcal{M}_{\text{FP}}] \exp\left(-S_{\text{m}}[\widehat{A}_i^a, \widehat{\Phi}^a] - S_{\text{quadr}}[a_i^a, \phi^a]\right). \tag{8}$$

Here, $S_{\text{m}}[\widehat{A}_i^a, \widehat{\Phi}^a]$ is the classical one-monopole action,

$$S_{\text{m}}[\widehat{A}_i^a, \widehat{\Phi}^a] = \int d^3x \left(\frac{1}{4} \widehat{F}_{ij}^a \widehat{F}_{ij}^a + \frac{1}{2} (\widehat{D}_i \widehat{\Phi})^a (\widehat{D}_i \widehat{\Phi})^a + \frac{\lambda}{8} (\widehat{\Phi}^{a2} - v^{a2})^2 \right), \tag{9}$$

and $S_{\text{quadr}}[a_i^a, \phi^a]$ that part of the full action which is quadratic in the quantum fluctuations:

$$S_{\text{quadr}}[a_i^a, \phi^a] = \frac{1}{2} \int d^3x \begin{pmatrix} a_i^a & \phi^a \end{pmatrix} \begin{pmatrix} \mathcal{M}_{ij}^{ab} & -\mathcal{M}_i^{ab} \\ \mathcal{M}_j^{ab} & \mathcal{M}^{ab} \end{pmatrix} \begin{pmatrix} a_j^b \\ \phi^b \end{pmatrix}, \tag{10}$$

where

$$\begin{aligned}\mathcal{M}_{ij}^{ab} &= -\widehat{D}_k^{ac} \widehat{D}_k^{cb} \delta_{ij} - 2e f^{acb} \widehat{F}_{ij}^c - e^2 f^{acd} f^{dc'b} \widehat{\Phi}^c \widehat{\Phi}^{c'} \delta_{ij}, \\ \mathcal{M}_i^{ab} &= 2e f^{acb} (\widehat{D}_i \widehat{\Phi})^c, \\ \mathcal{M}_j^{ab} &= 2e f^{acb} (\widehat{D}_j \widehat{\Phi})^c, \\ \mathcal{M}^{ab} &= -\widehat{D}_k^{ac} \widehat{D}_k^{cb} + \frac{\lambda}{2} (\widehat{\Phi}^{c2} - v^{c2}) \delta^{ab} + \lambda \widehat{\Phi}^a \widehat{\Phi}^b - e^2 f^{acd} f^{dc'b} \widehat{\Phi}^c \widehat{\Phi}^{c'}.\end{aligned}\tag{11}$$

Apart from the gauge zero modes which are eliminated by the gauge-fixing there are additional zero modes since the classical one-monopole solution violates

translational symmetry. The corresponding eigenfunctions of the quadratic form (11) are given by [6]

$$\begin{pmatrix} a_j^{(l)a} \\ \phi^{(l)a} \end{pmatrix} = \frac{1}{\sqrt{\mathcal{N}}} \begin{pmatrix} \hat{F}_{lj}^a \\ (\widehat{D}_l \hat{\Phi})^a \end{pmatrix},$$

$$\mathcal{N} = \int d^3x \left(\hat{F}_{ij}^a \hat{F}_{ij}^a + (\widehat{D}_i \hat{\Phi})^a (\widehat{D}_i \hat{\Phi})^a \right). \quad (12)$$

To eliminate these zero modes in the functional integral (8) we again use the standard Faddeev-Popov method of insertion of unity. The appropriate decomposition of unity is

$$\begin{aligned} 1 &= \det_{kl} \left[\frac{1}{\sqrt{\mathcal{N}}} \int d^3x \left(\frac{\delta}{\delta R_k} a_i^a \hat{F}_{li}^a + \frac{\delta}{\delta R_k} \phi^a (\widehat{D}_l \hat{\Phi})^a \right) \right] \\ &\times \int d\mathbf{R} \prod_{l=1}^3 \delta \left(\frac{1}{\sqrt{\mathcal{N}}} \int d^3x \left(a_i^a \hat{F}_{li}^a + \phi^a (\widehat{D}_l \hat{\Phi})^a \right) \right) \end{aligned} \quad (13)$$

where \mathbf{R} is the center of mass coordinate of the monopole. In the one-loop approximation the determinant in (13) can easily be evaluated and gives $\mathcal{N}^{3/2}$. Thus if we insert (13) in the functional integral (8) we obtain

$$\begin{aligned} Z_1 &= \int d\mathbf{R} \mathcal{N}^{3/2} \det [M_{\text{FP}}] \exp \left(-S_{\text{m}}[\hat{A}^a, \hat{\Phi}^a] \right) \\ &\times \int \mathcal{D}[a_i^a] \mathcal{D}[\phi^a] \prod_{l=1}^3 \delta \left(\frac{1}{\sqrt{\mathcal{N}}} \int d^3x \left(a_i^a \hat{F}_{li}^a + \phi^a (\widehat{D}_l \hat{\Phi})^a \right) \right) \\ &\times \exp \left(-S_{\text{quadr}}[a_i^a, \phi_i^a] \right). \end{aligned} \quad (14)$$

Since no further zero modes exist we can now do the functional integral over the quantum fluctuations. If we use Pauli-Villars regularization the expression for the one-monopole contribution to the functional integral in the one-loop approximation is given by

$$Z_1 = \int d\mathbf{R} M^3 \mathcal{N}^{3/2} \det [\mathcal{M}_{\text{FP}}] \left(\widetilde{\det} [\mathcal{M}] \right)^{-1/2} \exp \left(-S_{\text{m}}[\hat{A}_i^a, \hat{\Phi}^a] \right) \quad (15)$$

where M is the Pauli-Villars regulator mass and \mathcal{M} the quadratic form defined in (10) and (11). In case of the quantum fluctuation determinant we write $\widetilde{\det}$

instead of \det to indicate that the determinant has to be calculated with respect to the non-zero modes of \mathcal{M} only. Furthermore it is implied that both functional determinants are regularized by the Pauli-Villars method and normalized by the corresponding 'free' determinants, $\det[\mathcal{M}_{FP}^0]$ and $\det[\mathcal{M}^0]$. \mathcal{M}_{FP}^0 and \mathcal{M}^0 are obtained from \mathcal{M}_{FP} and \mathcal{M} by setting $\hat{A}_i^a = 0$ and $\hat{\Phi}^a = v^a$.

Finally, it is useful to introduce the dimensionless fields $\Phi^a \rightarrow v\Phi^a$ and $A_i^a \rightarrow vA_i^a$ as well as the coordinates $x_i \rightarrow x_i/(ev)$, $R_i \rightarrow R_i/(ev)$ and $M \rightarrow Mev$. In terms of these dimensionless quantities the functional integral (15) reads

$$Z_1 = \left(\frac{m_W}{e^2}\right)^{3/2} \int d\mathbf{R} M^3 \mathcal{N}^{3/2} \left(\frac{\det[\mathcal{M}_{FP}]}{\det[\mathcal{M}_{FP}^0]}\right) \left(\frac{\widetilde{\det}[\mathcal{M}]}{\det[\mathcal{M}^0]}\right)^{-1/2} \exp\left(-S_m[\hat{A}_i^a, \hat{\Phi}^a]\right) \quad (16)$$

where $m_W = ve$ is heavy vector boson mass,

$$S[\hat{A}, \hat{\Phi}] = \frac{m_W}{e^2} \int d^3x \left(\frac{1}{4} \hat{F}_{ij}^a \hat{F}_{ij}^a + \frac{1}{2} (\widehat{D}_i \hat{\Phi})^a (\widehat{D}_i \hat{\Phi})^a + \frac{\lambda}{8e^2} (\hat{\Phi}^{a2} - 1)^2 \right), \quad (17)$$

$$\hat{F}_{ij}^a = \partial_i \hat{A}_j^a - \partial_j \hat{A}_i^a + f^{abc} \hat{A}_i^b \hat{A}_j^c,$$

$$(\widehat{D}_i \hat{\Phi})^a = \partial_i \hat{\Phi}^a + f^{abc} \hat{A}_i^b \hat{\Phi}^c, \quad (18)$$

$$\mathcal{M}_{FP}^{ab} = \widehat{D}_i^{ac} \widehat{D}_i^{cb} + f^{acd} f^{dc'b} \hat{\Phi}^c \hat{\Phi}^{c'}, \quad (19)$$

and

$$\mathcal{M}_{ij}^{ab} = -\widehat{D}_k^{ac} \widehat{D}_k^{cb} \delta_{ij} - 2f^{acb} \hat{F}_{ij}^c - f^{acd} f^{dc'b} \hat{\Phi}^c \hat{\Phi}^{c'} \delta_{ij},$$

$$\mathcal{M}_i^{ab} = 2f^{acb} (\widehat{D}_i \hat{\Phi})^c,$$

$$\mathcal{M}_j^{ab} = 2f^{acb} (\widehat{D}_j \hat{\Phi})^c,$$

$$\mathcal{M}^{ab} = -\widehat{D}_k^{ac} \widehat{D}_k^{cb} + \frac{\lambda}{2e^2} (\hat{\Phi}^{c2} - 1) \delta^{ab} + \frac{\lambda}{e^2} \hat{\Phi}^a \hat{\Phi}^b - f^{acd} f^{dc'b} \hat{\Phi}^c \hat{\Phi}^{c'}. \quad (20)$$

With the help of the one-monopole result (16) one can now construct a formal expression for the partition function of a dilute gas of infinitely many monopoles

(see Section 4). However, since we are interested in an explicit expression for this partition function which then can be investigated numerically we are forced to work out the one-loop determinants. What we can say already at this point by only looking at the above formulas for the one-loop determinants is that the result will depend on the ratio λ/e^2 , only.

3 Calculation of the $SU(N)$ determinants

The method we are going to use to evaluate the determinants occurring in the expression for the one-monopole partition function is explained in detail in Refs. [13] and [14]. It is based on the observation that the ratio of two determinants, $\widetilde{\det\mathcal{A}}$ and $\widetilde{\det\mathcal{B}}$ with finite number of zero modes, $n_{\mathcal{A}}$ and $n_{\mathcal{B}}$, can be written as

$$\ln \left(\frac{\widetilde{\det\mathcal{A}}}{\widetilde{\det\mathcal{B}}} \right) = - \int_0^\infty \frac{dt}{t} \left[\text{Tr} \left(e^{-t\mathcal{A}} - e^{-t\mathcal{B}} \right) - n_{\mathcal{A}} + n_{\mathcal{B}} \right] . \quad (21)$$

In general this expression has to be regularized. The most suited regularization scheme is the ζ -function regularization [15] defined by

$$\begin{aligned} \ln \left(\frac{\widetilde{\det\mathcal{A}}}{\widetilde{\det\mathcal{B}}} \right)_{\text{reg}} &= \\ &= - \lim_{s \rightarrow 0} \left\{ \frac{d}{ds} \left(\frac{M^{2s}}{\Gamma(s)} \int_0^\infty dt t^{s-1} \left[\text{Tr} \left(e^{-t\mathcal{A}} - e^{-t\mathcal{B}} \right) - n_{\mathcal{A}} + n_{\mathcal{B}} \right] \right) \right\} , \end{aligned} \quad (22)$$

where M is again the Pauli-Villars regulator mass. The functional trace can easily be evaluated with respect to the complete set of plane waves $\exp(-ip_j x_j)$. Then we only have to shift all differential operators ∂_j occurring in \mathcal{A} and \mathcal{B} by ip_j and integrate over $d^d p$. Consequently (22) becomes

$$\begin{aligned} \ln \left(\frac{\widetilde{\det\mathcal{A}}}{\widetilde{\det\mathcal{B}}} \right)_{\text{reg}} &= - \lim_{s \rightarrow 0} \left\{ \frac{d}{ds} \left(\frac{M^{2s}}{\Gamma(s)} \int_0^\infty dt t^{s-1} \right. \right. \\ &\quad \times \left. \left[\int d^d x \int \frac{d^d p}{(2\pi)^d} \text{tr} \left(e^{-t\mathcal{A}(\partial_j \rightarrow \partial_j + ip_j)} - e^{-t\mathcal{B}(\partial_j \rightarrow \partial_j + ip_j)} \right) - n_{\mathcal{A}} + n_{\mathcal{B}} \right] \right) \right\} , \end{aligned} \quad (23)$$

where we used "tr" instead of "Tr" to indicate that the trace in the functional space has already been performed.

Mostly it is rather hopeless to compute expression (23) exactly and one has to rely on a certain approximation. The so-called "heat kernel" approximation is based on a power series expansion in t of the integrand occurring in (23). For dimensional reason an equivalent method is first to expand the exponential functions in powers of the covariant derivative (or the classical background-field) and then to integrate over the momentum. The approximation consists of cutting off the power series expansion in t at a suitable order and replacing the upper limit of the t -integration by a finite limit t_0 such that the integral is stationary at t_0 .

In Ref. [14] this method has been used to evaluate the determinants occurring in the functional integral of the pure gauge theory in the presence of an instanton background field. Thereby it turned out that already an approximation in which only the first two nontrivial terms of the expansion in t are taken into account agrees with the exact result within 3%. A possible explanation of the surprising accuracy of the approximation is that the first two nontrivial terms are already the third and fourth order terms in the t -expansion.

Since the evaluation of the determinants becomes much more complicated for the Yang-Mills-Higgs system than for the pure gauge theory we shall restrict ourselves to the first two nontrivial terms in the t -expansion, too. However, it should be mentioned that in our case these contributions will result from the second and third order terms of the t -expansion. Therefore we cannot expect a similar accuracy like in the corresponding instanton computation of [14] but we believe that the approximation should still be reasonable.

Let us begin with the evaluation of the Faddeev-Popov determinant, i.e.,

$$\ln \left(\frac{\det[-\mathcal{M}_{\text{FP}}]}{\det[-\mathcal{M}_{\text{FP}}^0]} \right)_{\text{reg}} = - \lim_{s \rightarrow 0} \left\{ \frac{d}{ds} \left(\frac{M^{2s}}{\Gamma(s)} \int_0^\infty dt t^{s-1} \text{Tr} \left(e^{t\mathcal{M}_{\text{FP}}} - e^{t\mathcal{M}_{\text{FP}}^0} \right) \right) \right\}. \quad (24)$$

As described above we insert a complete set of plane waves and expand the

exponential $\exp(t\mathcal{M}_{\text{FP}})$ in powers of the covariant derivatives. This leads us to

$$\begin{aligned}
\text{Tr} e^{t\mathcal{M}_{\text{FP}}} &= \text{tr} \int d^3x \int \frac{d^3p}{(2\pi)^3} e^{-p^2 t} \left\{ 1 + \left(\widehat{D}^2 + [\widehat{\Phi}]^2 \right) t - 4p_i p_j \widehat{D}_i \widehat{D}_j \frac{t^2}{2!} \right. \\
&\quad + \left(\widehat{D}^2 \left(\widehat{D}^2 + [\widehat{\Phi}]^2 \right) + [\widehat{\Phi}]^2 \left(\widehat{D}^2 + [\widehat{\Phi}]^2 \right) \right) \frac{t^2}{2!} \\
&\quad - 4p_i p_j \left(\left(\widehat{D}^2 + [\widehat{\Phi}]^2 \right) \widehat{D}_i \widehat{D}_j + \widehat{D}_i \left(\widehat{D}^2 + [\widehat{\Phi}]^2 \right) \widehat{D}_j \right. \\
&\quad \left. + \widehat{D}_i \widehat{D}_j \left(\widehat{D}^2 + [\widehat{\Phi}]^2 \right) \right) \frac{t^3}{3!} + 16p_i p_j p_k p_l \widehat{D}_i \widehat{D}_j \widehat{D}_k \widehat{D}_l \frac{t^4}{4!} \\
&\quad \left. + O(\widehat{D}^6) \right\} \\
&= \frac{\pi^{3/2}}{(2\pi)^3} \text{tr} \int d^3x \left\{ t^{-3/2} + [\widehat{\Phi}]^2 t^{-1/2} + \frac{1}{6} \left(-\widehat{D}_i \widehat{D}^2 \widehat{D}_i + \widehat{D}_i \widehat{D}_j \widehat{D}_i \widehat{D}_j \right. \right. \\
&\quad \left. \left. + \widehat{D}^2 [\widehat{\Phi}]^2 - 2\widehat{D}_i [\widehat{\Phi}]^2 \widehat{D}_i + [\widehat{\Phi}]^2 \widehat{D}^2 + 3[\widehat{\Phi}]^4 \right) t^{1/2} \right. \\
&\quad \left. + O(t^{3/2}) \right\}, \tag{25}
\end{aligned}$$

where, for simplicity, we already suppressed all terms with an odd number of p_i 's because they trivially vanish once the d^3p -integration is performed. Furthermore we introduced the shorthand notation $[X]$ which stands for

$$[X]^{ab} \equiv f^{acb} X^c \tag{26}$$

and should not be confused with the commutator notation $[\widehat{D}_i, \widehat{D}_j]$.

The evaluation of (25) goes straightforward. One easily verifies that

$$\text{tr} \left(\widehat{D}_i \widehat{D}_j \widehat{D}_i \widehat{D}_j - \widehat{D}_i \widehat{D}^2 \widehat{D}_i \right) = \frac{1}{2} \text{tr} \left([\widehat{D}_i, \widehat{D}_j] [\widehat{D}_i, \widehat{D}_j] \right) = -\frac{N}{2} \widehat{F}_{ij}^a \widehat{F}_{ij}^a, \tag{27}$$

and

$$\begin{aligned}
&\text{tr} \left(\widehat{D}^2 [\widehat{\Phi}] [\widehat{\Phi}] + [\widehat{\Phi}] [\widehat{\Phi}] \widehat{D}^2 - 2\widehat{D}_i [\widehat{\Phi}] [\widehat{\Phi}] \widehat{D}_i \right) = \\
&\quad \text{tr} \left([\widehat{D} \widehat{D} \widehat{\Phi}] [\widehat{\Phi}] + [\widehat{\Phi}] [\widehat{D} \widehat{D} \widehat{\Phi}] + 2[\widehat{D} \widehat{\Phi}] [\widehat{D} \widehat{\Phi}] \right) \\
&\quad = -2N \left((\widehat{D}_i \widehat{\Phi})^a (\widehat{D}_i \widehat{\Phi})^a + \frac{\lambda}{2e^2} (\widehat{\Phi}^{a2} - 1) \widehat{\Phi}^{b2} \right). \tag{28}
\end{aligned}$$

Substituting these expressions into (25) and performing the remaining traces we arrive at

$$\text{Tr} e^{t\mathcal{M}_{\text{FP}}} = \frac{N\pi^{3/2}}{(2\pi)^3} \int d^3x \left\{ \frac{N^2 - 1}{N} t^{-3/2} - \widehat{\Phi}^{a2} t^{-1/2} \right.$$

$$\begin{aligned}
& -\frac{1}{3} \left(\frac{1}{4} \hat{F}_{ij}^a \hat{F}_{ij}^a + (\widehat{D}_i \hat{\Phi})^a (\widehat{D}_i \hat{\Phi})^a + \frac{\lambda}{2e^2} (\hat{\Phi}^{a2} - 1) \hat{\Phi}^{b2} \right. \\
& \left. - \frac{3}{N} (\hat{\Phi}^{a2})^2 - \frac{3}{4} (d^{abc} \hat{\Phi}^b \hat{\Phi}^c)^2 \right) t^{1/2} + O(t^{3/2}) \Big\} . \quad (29)
\end{aligned}$$

The corresponding expression for the 'free' Faddeev-Popov matrix, $\mathcal{M}_{\text{FP}}^0$ is immediately obtained by setting $\hat{A}_i^a = 0$ and $\hat{\Phi}^a = \hat{v}^a = v^a/v$. Thus

$$\begin{aligned}
& \text{Tr} \left(e^{\mathcal{M}_{\text{FP}} t} - e^{\mathcal{M}_{\text{FP}}^0 t} \right) = \\
& = \frac{N\pi^{3/2}}{(2\pi)^3} \int d^3x \left\{ -(\hat{\Phi}^{a2} - 1) t^{-1/2} - \frac{1}{3} \left(\frac{1}{4} \hat{F}_{ij}^a \hat{F}_{ij}^a + (\widehat{D}_i \hat{\Phi})^a (\widehat{D}_i \hat{\Phi})^a \right. \right. \\
& \quad \left. \left. + \frac{\lambda}{2e^2} (\hat{\Phi}^{a2} - 1) \hat{\Phi}^{b2} - \frac{3}{N} \left((\hat{\Phi}^{a2})^2 - 1 \right) \right. \right. \\
& \quad \left. \left. - \frac{3}{4} \left((d^{abc} \hat{\Phi}^b \hat{\Phi}^c)^2 - (d^{abc} \hat{v}^b \hat{v}^c)^2 \right) \right) t^{1/2} + O(t^{3/2}) \right\} . \quad (30)
\end{aligned}$$

Since the operators \mathcal{M}_{FP} and $\mathcal{M}_{\text{FP}}^0$ are positive definite and possess a continuous spectrum we can employ the heat kernel approximation. Thereby we assume that the left-hand side of (30) is a rapidly decaying function of t which can be approximated by the first few terms of its expansion. Thus, to obtain an estimate for the t -integral in (25) we insert the expansion (30) into (25) and replace the infinite upper limit of the integral by a finite limit t_0 . For t_0 we choose that value at which the result as a function of t_0 possesses an extremum. In this way we obtain

$$\ln \left(\frac{\det[-\mathcal{M}_{\text{FP}}]}{\det[-\mathcal{M}_{\text{FP}}^0]} \right)_{\text{reg}} \approx - \lim_{s \rightarrow 0} \left\{ \frac{d}{ds} \left(\frac{M^{2s}}{\Gamma(s)} \int_0^{t_0} dt t^{s-1} \left[\alpha_{\mathcal{M}_{\text{FP}}} t^{-1/2} - \beta_{\mathcal{M}_{\text{FP}}} t^{1/2} \right] \right) \right\} , \quad (31)$$

where

$$\begin{aligned}
\alpha_{\mathcal{M}_{\text{FP}}} &= \frac{N\pi^{3/2}}{(2\pi)^3} \int d^3x \left(1 - \hat{\Phi}^{a2} \right) , \\
\beta_{\mathcal{M}_{\text{FP}}} &= \frac{N\pi^{3/2}}{3(2\pi)^3} \int d^3x \left(\frac{1}{4} \hat{F}_{ij}^a \hat{F}_{ij}^a + (\widehat{D}_i \hat{\Phi})^a (\widehat{D}_i \hat{\Phi})^a + \frac{\lambda}{2e^2} (\hat{\Phi}^{a2} - 1) \hat{\Phi}^{b2} \right. \\
& \quad \left. - \frac{3}{N} \left((\hat{\Phi}^{a2})^2 - 1 \right) - \frac{3}{4} \left((d^{abc} \hat{\Phi}^b \hat{\Phi}^c)^2 - (d^{abc} \hat{v}^b \hat{v}^c)^2 \right) \right) \quad (32)
\end{aligned}$$

and t_0 , determined by the extremum condition $\alpha_{\mathcal{M}_{\text{FP}}} t_0^{-1/2} - \beta_{\mathcal{M}_{\text{FP}}} t_0^{1/2} = 0$ is given by

$$t_0 = \frac{\alpha_{\mathcal{M}_{\text{FP}}}}{\beta_{\mathcal{M}_{\text{FP}}}}. \quad (33)$$

Note, that t_0 must be positive. Otherwise we must take into account the next higher order in the t -expansion. In fact, we can easily convince ourselves that t_0 indeed is always positive. Since $\hat{\Phi}^{a2} \leq 1$ the coefficient $\alpha_{\mathcal{M}_{\text{FP}}}$ is clearly positive. As far as $\beta_{\mathcal{M}_{\text{FP}}}$ is concerned the only negative term is the third one which, according to the field equation for $\hat{\Phi}$ will exactly be canceled by the second term. Consequently, both coefficients are always positive.

Finally we have to perform the integral over dt and to take the limit $s \rightarrow 0$ which leads to the following result:

$$\ln \left(\frac{\det[-\mathcal{M}_{\text{FP}}]}{\det[-\mathcal{M}_{\text{FP}}^0]} \right)_{\text{reg}} \approx 4\sqrt{\alpha_{\mathcal{M}_{\text{FP}}}\beta_{\mathcal{M}_{\text{FP}}}} = 4\beta_{\mathcal{M}_{\text{FP}}} t_0^{1/2}. \quad (34)$$

Apart from the fact that the expressions become rather lengthy the computation of the quantum fluctuation determinant, i.e.,

$$\ln \left(\frac{\widetilde{\det}[\mathcal{M}]}{\det[\mathcal{M}^0]} \right)_{\text{reg}} = -\lim_{s \rightarrow 0} \left\{ \frac{d}{ds} \left(\frac{M^{2s}}{\Gamma(s)} \int_0^\infty dt t^{s-1} \text{Tr} \left(e^{t\mathcal{M}} - e^{t\mathcal{M}^0} - n_{\mathcal{M}} \right) \right) \right\}, \quad (35)$$

goes exactly the same way. Instead of (25) one finds

$$\begin{aligned} \text{Tre}^{\mathcal{M}t} &= \\ &= \text{tr} \int d^3x \int \frac{d^3p}{(2\pi)^3} e^{-p^2 t} \{ (\delta_{ij} + 1) \\ &\quad + \left((\widehat{D}^2 + [\hat{\Phi}]^2) \delta_{ij} + 2[\hat{F}_{ij}] + \widehat{D}^2 - V(\hat{\Phi}) + [\hat{\Phi}]^2 \right) t \\ &\quad - 4p_k p_l \widehat{D}_k \widehat{D}_l (\delta_{ij} + 1) \frac{t^2}{2!} + \left((\widehat{D}^2 + [\hat{\Phi}]^2)^2 \delta_{ij} + 2[\hat{F}_{ij}] (\widehat{D}^2 + [\hat{\Phi}]^2) \right. \\ &\quad \left. + 2(\widehat{D}^2 + [\hat{\Phi}]^2) [\hat{F}_{ij}] + 4[\hat{F}_{ik}] [\hat{F}_{kj}] - 4[\widehat{D}_i \hat{\Phi}] [\widehat{D}_j \hat{\Phi}] - 4[\widehat{D}_k \hat{\Phi}]^2 \right. \\ &\quad \left. + (\widehat{D}^2 - V(\hat{\Phi}) + [\hat{\Phi}]^2)^2 \right) \frac{t^2}{2!} - 4p_k p_l \left(\widehat{D}_k \widehat{D}_l (\widehat{D}^2 + [\hat{\Phi}]^2) \delta_{ij} \right. \\ &\quad \left. + \widehat{D}_k (\widehat{D}^2 + [\hat{\Phi}]^2) \widehat{D}_l \delta_{ij} + (\widehat{D}^2 + [\hat{\Phi}]^2) \widehat{D}_k \widehat{D}_l \delta_{ij} + 2\widehat{D}_k \widehat{D}_l [\hat{F}_{ij}] \right. \\ &\quad \left. + 2\widehat{D}_k [\hat{F}_{ij}] \widehat{D}_l + 2[\hat{F}_{ij}] \widehat{D}_k \widehat{D}_l + \widehat{D}_k \widehat{D}_l (\widehat{D}^2 - V(\hat{\Phi}) + [\hat{\Phi}]^2) \right. \\ &\quad \left. + \widehat{D}_k (\widehat{D}^2 - V(\hat{\Phi}) + [\hat{\Phi}]^2) \widehat{D}_l + (\widehat{D}^2 - V(\hat{\Phi}) + [\hat{\Phi}]^2) \widehat{D}_k \widehat{D}_l \right) \frac{t^3}{3!} \end{aligned}$$

$$\begin{aligned}
& +16p_k p_l p_m p_n \widehat{D}_k \widehat{D}_l \widehat{D}_m \widehat{D}_n (\delta_{ij} + 1) \frac{t^4}{4!} + O(\widehat{D}^6) \Big\} \\
& = \frac{\pi^{3/2}}{(2\pi)^3} \text{tr} \int d^3x \left\{ 4t^{-3/2} + \left(4[\widehat{\Phi}]^2 - V(\widehat{\Phi}) \right) t^{-1/2} \right. \\
& \quad + \frac{1}{6} \left(4 \left(\widehat{D}_i \widehat{D}_j \widehat{D}_i \widehat{D}_j - \widehat{D}_i \widehat{D}^2 \widehat{D}_i \right) - 12[\widehat{F}_{ij}][\widehat{F}_{ij}] - 24[\widehat{D}_i \widehat{\Phi}][\widehat{D}_i \widehat{\Phi}] \right. \\
& \quad \quad + 4 \left(\widehat{D}^2 [\widehat{\Phi}]^2 - 2\widehat{D}_i [\widehat{\Phi}]^2 \widehat{D}_i + [\widehat{\Phi}]^2 \widehat{D}^2 + 3[\widehat{\Phi}]^4 \right) \\
& \quad \quad - \left(\widehat{D}^2 V(\widehat{\Phi}) - 2\widehat{D}_i V(\widehat{\Phi}) \widehat{D}_i + V(\widehat{\Phi}) \widehat{D}^2 - 3V(\widehat{\Phi})^2 \right) \\
& \quad \quad \left. \left. - 3 \left([\widehat{\Phi}]^2 V(\widehat{\Phi}) + V(\widehat{\Phi}) [\widehat{\Phi}]^2 \right) \right) t^{1/2} + O(t^{3/2}) \right\} , \tag{36}
\end{aligned}$$

where, in order to simplify the expression at least a little bit, we used the short-hand notation $V(\widehat{\Phi})$ for

$$V^{ab}(\widehat{\Phi}) \equiv \frac{\lambda}{2e^2} \left(\widehat{\Phi}^{c2} - 1 \right) \delta^{ab} + \frac{\lambda}{e^2} \widehat{\Phi}^a \widehat{\Phi}^b . \tag{37}$$

With the help of (27), (28) and

$$\begin{aligned}
& \text{tr} \left(\widehat{D}^2 V(\widehat{\Phi}) + V(\widehat{\Phi}) \widehat{D}^2 - 2\widehat{D}_i V(\widehat{\Phi}) \widehat{D}_i \right) = \\
& = \frac{(N^2 + 1) \lambda}{e^2} \left((\widehat{D}_i \widehat{\Phi})^a (\widehat{D}_i \widehat{\Phi})^a + \widehat{\Phi}^a (\widehat{D}_i \widehat{D}_i \widehat{\Phi})^a \right) \\
& = \frac{(N^2 + 1) \lambda}{e^2} \left((\widehat{D}_i \widehat{\Phi})^a (\widehat{D}_i \widehat{\Phi})^a + \frac{\lambda}{2e^2} \left(\widehat{\Phi}^{a2} - 1 \right) \widehat{\Phi}^{b2} \right) \tag{38}
\end{aligned}$$

we obtain from (36)

$$\begin{aligned}
& \text{Tre}^{\mathcal{M}t} = \\
& = \frac{\pi^{3/2}}{(2\pi)^3} \int d^3x \left\{ 4 \left(N^2 - 1 \right) t^{-3/2} \right. \\
& \quad - \left(4N \widehat{\Phi}^{a2} + \frac{\lambda}{2e^2} \left(\widehat{\Phi}^{a2} - 1 \right) \left(N^2 - 1 \right) + \frac{\lambda}{e^2} \widehat{\Phi}^{a2} \right) t^{-1/2} \\
& \quad + \frac{1}{6} \left(10N \widehat{F}_{ij}^a \widehat{F}_{ij}^a + 16N (\widehat{D}_i \widehat{\Phi})^a (\widehat{D}_i \widehat{\Phi})^a - \frac{N\lambda}{e^2} \left(\widehat{\Phi}^{a2} - 1 \right) \widehat{\Phi}^{b2} \right. \\
& \quad \quad - (N^2 + 1) \frac{\lambda}{e^2} (\widehat{D}_i \widehat{\Phi})^a (\widehat{D}_i \widehat{\Phi})^a + 24 \left(\widehat{\Phi}^{a2} \right)^2 + 6N \left(d^{abc} \widehat{\Phi}^b \widehat{\Phi}^c \right)^2 \\
& \quad \quad + \frac{\lambda^2}{4e^4} \left(\left(N^2 - 1 \right) \left(\widehat{\Phi}^{a2} - 1 \right) \left(\widehat{\Phi}^{b2} - 3 \right) \right. \\
& \quad \quad \quad \left. \left. + 8 \left(\widehat{\Phi}^{a2} - 1 \right) \widehat{\Phi}^{b2} + 12 \left(\widehat{\Phi}^{a2} \right)^2 \right) \right) t^{1/2} + O(t^{3/2}) \Big\} . \tag{39}
\end{aligned}$$

If we substitute this expression and the corresponding one for \mathcal{M}^0 into (35) and remember that \mathcal{M} has $n_{\mathcal{M}} = 3$ translational zero modes we arrive at

$$\ln \left(\frac{\widetilde{\det}[\mathcal{M}]}{\det[\mathcal{M}^0]} \right)_{\text{reg}} \approx - \lim_{s \rightarrow 0} \left\{ \frac{d}{ds} \left(\frac{M^{2s}}{\Gamma(s)} \int_0^{t_1} dt t^{s-1} [\alpha_{\mathcal{M}} t^{-1/2} + \beta_{\mathcal{M}} t^{1/2} - 3] \right) \right\}, \quad (40)$$

where

$$\begin{aligned} \alpha_{\mathcal{M}} &= \frac{\pi^{3/2}}{(2\pi)^3} \left(4N + \frac{\lambda}{2e^2} (N^2 + 1) \right) \int d^3x (1 - \hat{\Phi}^{a2}), \\ \beta_{\mathcal{M}} &= \frac{\pi^{3/2}}{6(2\pi)^3} \int d^3x \left(10N \hat{F}_{ij}^a \hat{F}_{ij}^a + 16N (\widehat{D}_i \hat{\Phi})^a (\widehat{D}_i \hat{\Phi})^a - \frac{N\lambda}{e^2} (\hat{\Phi}^{a2} - 1) \hat{\Phi}^{b2} \right. \\ &\quad \left. - (N^2 + 1) \frac{\lambda}{e^2} (\widehat{D}_i \hat{\Phi})^a (\widehat{D}_i \hat{\Phi})^a + 24 \left((\hat{\Phi}^{a2})^2 - 1 \right) \right. \\ &\quad \left. + \frac{\lambda^2}{4e^4} (\hat{\Phi}^{a2} - 1) \left((N^2 - 1) (\hat{\Phi}^{b2} - 3) + 4 (5\hat{\Phi}^{b2} + 3) \right) \right. \\ &\quad \left. + 6N \left((d^{abc} \hat{\Phi}^b \hat{\Phi}^c)^2 - (d^{abc} \hat{v}^b \hat{v}^c)^2 \right) \right). \end{aligned} \quad (41)$$

The expression for t_1 follows from the extremum condition $\alpha_{\mathcal{M}} t_1^{-1/2} + \beta_{\mathcal{M}} t_1^{1/2} - 3 = 0$. Since $\hat{\Phi}^2 \leq 1$ the coefficient $\alpha_{\mathcal{M}}$ is always positive and therefore

$$t_1^{1/2} = \begin{cases} \frac{3}{2\beta_{\mathcal{M}}} + \sqrt{\frac{9}{4\beta_{\mathcal{M}}^2} - \frac{\alpha_{\mathcal{M}}}{\beta_{\mathcal{M}}}}, & \text{if } \beta_{\mathcal{M}} < 0; \\ \frac{3}{2\beta_{\mathcal{M}}} - \sqrt{\frac{9}{4\beta_{\mathcal{M}}^2} - \frac{\alpha_{\mathcal{M}}}{\beta_{\mathcal{M}}}}, & \text{if } \beta_{\mathcal{M}} > 0. \end{cases} \quad (42)$$

Since $t_1^{1/2}$ must be real and positive $\alpha_{\mathcal{M}}$ and $\beta_{\mathcal{M}}$ must satisfy the inequality $\alpha_{\mathcal{M}} \beta_{\mathcal{M}} \leq 9/4$. Otherwise the next higher order in the t -expansion must be taken into account. For $N = 2$, i.e., for the $SU(2)$ gauge model, we have checked explicitly that the inequality holds. But so far we do not have a general proof that this inequality holds for monopole configurations of arbitrary $SU(N)$ gauge groups.

Under the assumption that the above inequality holds for arbitrary N we can integrate over dt and take the limit $s \rightarrow 0$. This then gives

$$\ln \left(\frac{\widetilde{\det}[\mathcal{M}]}{\det[\mathcal{M}^0]} \right)_{\text{reg}} \approx 3\gamma_E + 2\alpha_{\mathcal{M}} t_1^{-1/2} - 2\beta_{\mathcal{M}} t_1^{1/2} + 3 \ln (M^2 t_1). \quad (43)$$

where $\gamma_E \approx 0.5772\dots$ is the Euler constant.

If we insert (34) and (43) into (16) we obtain the following expression for the one-monopole partition function

$$\begin{aligned}
Z_1 &= \left(\frac{m_W}{e^2} \right)^{3/2} \int d\mathbf{R} M^3 \mathcal{N}^{3/2} \exp \left(-S_m[\hat{A}_i^a, \hat{\Phi}^a] \right) \\
&\quad \times \exp \left[4\beta_{\mathcal{M}_{\text{FP}}} t_0^{1/2} \right] \exp \left[\frac{3}{2} \gamma_E - \alpha_{\mathcal{M}} t_1^{-1/2} + \beta_{\mathcal{M}} t_1^{1/2} - \frac{3}{2} \ln(M^2 t_1) \right] \\
&= \left(\frac{m_W}{e^2/4\pi} \right)^{3/2} \int d\mathbf{R} \frac{1}{2\pi} A(\lambda/e^2) \exp \left[-\frac{m_W}{e^2/4\pi} C(\lambda/e^2) \right], \tag{44}
\end{aligned}$$

where

$$C(\lambda/e^2) = \left(\frac{m_W}{e^2/4\pi} \right)^{-1} S_m[\hat{A}_i^a, \hat{\Phi}^a] \tag{45}$$

is the usual one-monopole mass integral and

$$A(\lambda/e^2) = 2\pi \left(\frac{\mathcal{N}}{4\pi} \right)^{3/2} \frac{\exp \left[4\beta_{\mathcal{M}_{\text{FP}}} t_0^{1/2} + \beta_{\mathcal{M}} t_1^{1/2} - \alpha_{\mathcal{M}} t_1^{-1/2} - \frac{3}{2} \gamma_E \right]}{t_1^{3/2}}. \tag{46}$$

Note, that in contrast to the corresponding result for the one-loop instanton partition function in four dimensions [14] the Pauli-Villars regulator mass M drops out from the final expression.

Having an explicit expression for one-monopole partition function at hand we can now study the dilute monopole gas. The simplest example of such a gas is the $SU(2)$ monopole gas consisting of 't Hooft-Polyakov monopoles [6].

4 The $SU(2)$ dilute monopole gas

For the $SU(2)$ gauge group and $\lambda/e^2 > 0$ it has been shown that if the mean distance between two widely separated monopoles ($|\mathbf{R}_i - \mathbf{R}_j| \gg 1$) is large compared to $(\lambda/e^2)^{-1}$ the superposition principle holds and the interaction between the monopoles is described by the Coulomb interaction [16]. Thus in this case the action of a system of n monopoles with magnetic charges $q_i = \pm 1$ can be written in the form

$$S_n = S_m \sum_i q_i^2 + \frac{\pi m_W}{2e^2} \sum_{i \neq j} \frac{q_i q_j}{|\mathbf{R}_i - \mathbf{R}_j|} \tag{47}$$

and the partition function Z for a gas of monopoles becomes

$$Z = \sum_n \frac{(z_1)^n}{n!} \int \prod_{k=1}^n d\mathbf{R}_k \exp \left(-\frac{\pi m_W}{2e^2} \sum_{i \neq j} \frac{q_i q_j}{|\mathbf{R}_i - \mathbf{R}_j|} \right) \quad (48)$$

with (cf. (44))

$$z_1 = \frac{1}{2\pi} \left(\frac{m_W}{(e^2/4\pi)} \right)^{3/2} A(\lambda/e^2) \exp \left[-\frac{m_W}{e^2/4\pi} C(\lambda/e^2) \right]. \quad (49)$$

Note, that in the Prasad-Sommerfield limit $\lambda/e^2 \rightarrow 0$ there appears a change in the interaction of monopoles [17, 18, 19]. The Higgs field becomes massless, and the attractive force associated with it becomes long range. This force cancels the repulsive magnetic force between like charges and doubles the attractive force between unlike charges. Consequently two Prasad-Sommerfield monopoles do not interact via the Coulomb force if they possess the same magnetic charge.

The Coulomb gas of magnetic monopoles interacting with an external magnetic charge $\rho(\mathbf{x})$ is described by the partition function

$$\begin{aligned} Z[\eta] = \sum_n \frac{(z_1)^n}{n!} \int \prod_{k=1}^n d\mathbf{R}_k \exp \left(-\frac{\pi m_W}{2e^2} \sum_{i \neq j} \frac{q_i q_j}{|\mathbf{R}_i - \mathbf{R}_j|} \right) \\ \times \exp \left(i \int d^3x \sum_i q_i \delta(\mathbf{x} - \mathbf{R}_i) \eta(\mathbf{x}) \right) \end{aligned} \quad (50)$$

where η is the potential corresponding to the external magnetic charge, i.e., $\Delta\eta(\mathbf{x}) = 2\pi\rho(\mathbf{x})$. A path integral expression for $Z[\eta]$ is now easily obtained by rewriting the Coulomb interaction in (50) as an integral over an auxiliary field $\chi(\mathbf{R})$. The resulting expression reads

$$Z[\eta] = \int \mathcal{D}[\chi] \exp \left[-\frac{e^2}{4\pi^2 m_W} \int d^3x \left(\frac{1}{2} (\nabla(\chi - \eta))^2 - \omega^2 \cos \chi \right) \right] \quad (51)$$

where ω given by

$$\omega^2 = 8\pi^2 \frac{m_W}{e^2} z_1 = \left(\frac{m_W}{(e^2/4\pi)} \right)^{5/2} A(\lambda/e^2) \exp \left[-\frac{m_W}{e^2/4\pi} C(\lambda/e^2) \right] \quad (52)$$

is the gauge boson mass in the unbroken $U(1)$ gauge sector in units of m_W , that is, $m_{U(1)} = \omega m_W$.

The Euclidean functional integral (51) describes a Debye plasma of monopoles and antimonopoles if the Debye radius given by $1/m_{U(1)}$ is large compared to the mean distance between the monopoles. If n is the density of monopoles the mean distance is proportional to $n^{-1/3}$. The density of monopoles on the other hand is proportional to $(e m_{U(1)})^2$. Thus the condition for the Debye approximation is

$$\frac{m_{U(1)}}{e^2} \ll 1. \quad (53)$$

Since $m_{U(1)}$ is proportional to $\exp[-m_W C(\lambda/e^2)/2(e^2/4\pi)]$ it is clear that by taking $(e^2/4\pi)$ small the monopole gas can be made as dilute as one likes. The nonlinearities in functional integral (51) become exponentially small, too, because $m_{U(1)}/e^2$ is nothing but the dimensionless effective coupling of the monopole gas. However, because of the long-range magnetic Coulomb interaction between monopoles, the gas does not become noninteracting even for very small densities. This is a general feature of the three dimensional compact $U(1)$ gauge model.

The "area law" behavior follows from the partition function $Z[\eta]$ if one considers an external monopole charge density $\rho_l(\mathbf{x})$ generated by an Wilson loop l in the x_1, x_2 plane. For a large Wilson loop l $\rho_l(\mathbf{x})$ is approximately given by the monopole charge density which is generated by an classical external electric current loop $j_i = \epsilon_{ijk} \partial_j H_k$ with $\partial_j H_j = \rho_l$, i.e.,

$$\rho_l(\mathbf{x}) = \delta'(x_3) \Theta_l(x_1, x_2), \quad (54)$$

where $\Theta(x_1, x_2) = 1$ if x_1 and x_2 are the coordinates of a point inside the loop l and zero otherwise.

In the semi-classical approximation $Z[\eta_l]$ is approximated by

$$Z[\eta_l] \approx \int \mathcal{D}[\chi] \exp \left[-\frac{e^2}{4\pi^2 m_W} \int d^3x \left(\frac{1}{2} (\nabla(\chi_{\text{cl}} - \eta_l))^2 - \omega^2 \cos \chi_{\text{cl}} \right) \right], \quad (55)$$

where χ_{cl} is a solution of

$$\nabla^2 \chi_{\text{cl}} = \nabla^2 \eta_l + \omega^2 \sin \chi_{\text{cl}} = 2\pi \rho_l + \omega^2 \sin \chi_{\text{cl}}. \quad (56)$$

An approximate solution of (56) with ρ_l given by (54) is

$$\chi_{\text{cl}} = \begin{cases} 4\Theta_l(x_1, x_2) \arctan(e^{-\omega x_3}), & \text{if } x_3 > 0, \\ 4\Theta_l(x_1, x_2) \arctan(e^{\omega x_3}), & \text{if } x_3 < 0. \end{cases} \quad (57)$$

If we insert this solution into (55) we finally arrive at

$$Z[\eta_l] \approx e^{-\sigma S_l} \quad (58)$$

where S_l is the area enclosed by the loop l and

$$\sigma = \frac{2e^2}{\pi^2} m_W \omega = \frac{2e^2}{\pi^2} m_{U(1)} \quad (59)$$

the string tension. This is Polyakov's formula (corrected for misprints).

To summarize, the polarization of a dilute monopole gas of 't Hooft-Polyakov monopoles screens the long range magnetic Coulomb force so that an infrared stable semiclassical expansion becomes possible. There are no massless particles in the theory and non-singlet states are confined by a linearly rising potential. The $U(1)$ gauge boson mass and the string tension of the dilute gas are given by

$$\frac{m_{U(1)}}{m_W} = \omega \left(\frac{m_W}{e^2/4\pi}, \frac{\lambda}{e^2} \right), \quad (60)$$

and

$$\frac{\sigma}{m_W^2} = 8\pi \left(\frac{m_W}{e^2/4\pi} \right)^{-1} \omega \left(\frac{m_W}{e^2/4\pi}, \frac{\lambda}{e^2} \right), \quad (61)$$

where

$$\omega^2 \left(\frac{m_W}{e^2/4\pi}, \frac{\lambda}{e^2} \right) = \left(\frac{m_W}{(e^2/4\pi)} \right)^{5/2} A(\lambda/e^2) \exp \left[-\frac{m_W}{e^2/4\pi} C(\lambda/e^2) \right]. \quad (62)$$

5 Numerical results for the $SU(2)$ gauge model

The monopole configuration of the $SU(2)$ gauge model is of the form [1]

$$\begin{aligned} \vec{A}^a(\vec{r})\sigma^a &= \vec{\sigma} \times \vec{r} \frac{h(r)}{r}, \\ \hat{\Phi}^a(\vec{r})\sigma^a &= \vec{r} \cdot \vec{\sigma} \frac{g(r)}{r}, \end{aligned} \quad (63)$$

where σ^a are the Pauli matrices. The functions $h(r)$ and $g(r)$ are obtained by minimizing the action (17), that is the monopole mass integral

$$C(\lambda/e^2) = \int_0^\infty dr \left((rh'(r) + h(r))^2 + 2h^2(r) \left(1 + \frac{rh(r)}{2} \right)^2 + \frac{r^2}{2} g'^2(r) + g^2(r) (1 + rh(r))^2 + \frac{\lambda}{8e^2} r^2 (g^2(r) - 1)^2 \right). \quad (64)$$

The corresponding differential equations are

$$\begin{aligned} r^2 h''(r) + 2rh'(r) - 2h(r) - rg^2(r)(1 + rh(r)) - 3rh^2(r) - r^2 h^3(r) &= 0, \\ r^2 g''(r) + 2rg'(r) - 2g(r) + \frac{\lambda}{2e^2} r^2 g(r) - 4rh(r)g(r) \\ - 2r^2 h^2(r)g(r) - \frac{\lambda}{2e^2} r^2 g^3(r) &= 0, \end{aligned} \quad (65)$$

which apart from the Prasad-Sommerfield limit ($\lambda/e^2 \rightarrow 0$) [20] can only be solved numerically.

To determine the functions $h(r)$ and $g(r)$ we use instead of (65) the corresponding system of coupled nonlinear integral equations derived by Bais and Primack [21] which can be solved numerically by simple iteration. In terms of $h(r)$ and $f(r) = 1 - g(r)$ these integral equations are given by

$$\begin{aligned} h(r) &= h_0(r) + \int_0^\infty G_h(r, r') (A_h(r') + B[h_0(r')]) dr', \\ f(r) &= g_0(r) + \int_0^\infty G_f(r, r') (A_f(r') + B[f_0(r')]) dr', \end{aligned} \quad (66)$$

where

$$G_h(r, r') = \begin{cases} \left(\frac{\cosh(r)}{r} - \frac{\sinh(r)}{r^2} \right) \frac{e^{-r'}(1+1/r')}{r'}, & \text{if } r' > r, \\ \left(\frac{\cosh(r')}{r'} - \frac{\sinh(r')}{r'^2} \right) \frac{e^{-r}(1+1/r)}{r}, & \text{if } r' < r, \end{cases}$$

$$G_f(r, r') = \begin{cases} \left(\frac{\cosh(\sqrt{\lambda/e^2}r)}{\sqrt{\lambda/e^2}r} - \frac{\sinh(\sqrt{\lambda/e^2}r)}{(\sqrt{\lambda/e^2}r)^2} \right) \frac{e^{-\sqrt{\lambda/e^2}r'} \left(1 + 1/(\sqrt{\lambda/e^2}r') \right)}{\sqrt{\lambda/e^2}r'}, & \text{if } r' > r, \\ \left(\frac{\cosh(\sqrt{\lambda/e^2}r')}{\sqrt{\lambda/e^2}r'} - \frac{\sinh(\sqrt{\lambda/e^2}r')}{(\sqrt{\lambda/e^2}r')^2} \right) \frac{e^{-\sqrt{\lambda/e^2}r} \left(1 + 1/(\sqrt{\lambda/e^2}r) \right)}{\sqrt{\lambda/e^2}r}, & \text{if } r' < r, \end{cases} \quad (67)$$

are the Green's functions of the homogeneous equations corresponding to (65),

$$\begin{aligned} A_h(r) &= -r(1 + rh(r))(f^2(r) - 2f(r)) - 3rh^2(r) - r^2h^3(r) - r, \\ A_f(r) &= 2 + 2rh(r)(2 + rh(r))(1 - f(r)) + \frac{\lambda}{2e^2}r^2f^2(r)(3 - f(r)), \end{aligned} \quad (68)$$

are the nonlinear sources and

$$\begin{aligned} B[h_0(r)] &= r^2h_0''(r) + 2rh_0'(r) - (2 + r^2)h_0(r), \\ B[f_0(r)] &= r^2f_0''(r) + 2rf_0'(r) - \left(2 + \frac{\lambda}{e^2}r^2 \right) f_0(r). \end{aligned} \quad (69)$$

The functions $h_0(r)$ and $f_0(r)$ must satisfy the boundary conditions

$$\begin{aligned} f(r) &\xrightarrow{r \rightarrow 0} 1 - c_f(\lambda/e^2)r + O(r^3), & h(r) &\xrightarrow{r \rightarrow 0} c_h(\lambda/e^2)r + O(r^3), \\ f(r) &\xrightarrow{r \rightarrow \infty} 0, & h(r) &\xrightarrow{r \rightarrow \infty} -1/r, \end{aligned} \quad (70)$$

but can otherwise arbitrarily be chosen. As in [21] we have used

$$h_0(r) = -\frac{r}{r^2 + b(\lambda/e^2)} \quad \text{and} \quad f_0(r) = 1 - \frac{r}{(r^2 + a(\lambda/e^2))^{1/2}}, \quad (71)$$

where $a(\lambda/e^2)$ and $b(\lambda/e^2)$ are to be determined by minimizing the monopole mass integral $C(\lambda/e^2)$. The functionals $B[h_0(r)]$ and $B[f_0(r)]$ then take the form

$$\begin{aligned} B[h_0(r)] &= \frac{2b(\lambda/e^2)r(3r^2 - b(\lambda/e^2))}{(r^2 + b(\lambda/e^2))^3} - (2 + r^2)h_0(r) \\ B[f_0(r)] &= \frac{a(\lambda/e^2)r(r^2 - 2a)}{(r^2 + a(\lambda/e^2))^{5/2}} - \left(\frac{\lambda}{e^2}r^2 + 2 \right) f_0(r). \end{aligned} \quad (72)$$

For a detailed derivation of the coupled set of integral equations (66) we refer the reader to ref. [21].

To study the λ/e^2 -dependence of the $U(1)$ gauge boson mass and the string tension σ we have calculated iterative solutions of the system (66) for various values of λ/e^2 . The results for the mass integral $C(\lambda/e^2)$ and $A(\lambda/e^2)$ which together determine the total λ/e^2 -dependence of $m_{U(1)}$ and σ are plotted in Fig. 1 and Fig. 2. In addition we have listed a few illustrative values in Table 1.

λ/e^2	0.1	0.5	2.0	10.0	100.0
$C(\lambda/e^2)$	1.106	1.119	1.291	1.433	1.617
$A(\lambda/e^2)$	$3.7 \cdot 10^{-19}$	$9.2 \cdot 10^{-3}$	6.576	$1.74 \cdot 10^{-1}$	$1.14 \cdot 10^{-10}$

Table I. $C(\lambda/e^2)$ and $A(\lambda/e^2)$ for different values of λ/e^2

In contrast to the mass integral $C(\lambda/e^2)$ which is a rather slowly increasing function of λ/e^2 between 1.0 ($\lambda/e^2 = 0$) and 1.787 ($\lambda/e^2 \rightarrow \infty$) the function $A(\lambda/e^2)$ varies substantially. This strong λ/e^2 dependence can formally be understood if one compares the various terms which contribute to mass integral $C(\lambda/e^2)$ and the coefficients $\alpha_{M_{\text{FP}}}$, $\beta_{M_{\text{FP}}}$, α_M and β_M in the heat kernel expansion. Expressions that do appear in these coefficients but not in the mass integral are for example $\int d^3x(1 - \hat{\Phi}^2)$ and $\int d^3x((\hat{\Phi}^2)^2 - 1)$. In contrast to the similar term $(\lambda/e^2) \int d^3x(\hat{\Phi}^2 - 1)^2$ which appears in the mass integral the size of these expressions depends very much on the shape of Higgs field which is determined by the ratio λ/e^2 .

At $\lambda/e^2 \approx 2.2$ the function $A(\lambda/e^2)$ has a maximum. Since on the other hand $\lambda/e^2 = m_{\text{Higgs}}^2/m_W^2$ this result is equivalent with the statement that $A(\lambda/e^2)$ is largest if the square of the Higgs mass is roughly twice as large as the square of the heavy vector boson mass. For $\lambda/e^2 > 2.2$ $A(\lambda/e^2)$ decreases again and seems to vanish in the limit $\lambda/e^2 \rightarrow \infty$.

The behavior of $A(\lambda/e^2)$ for small and large values of λ/e^2 becomes more transparent in Fig. 3 where the logarithm of $A(\lambda/e^2)$ is shown. For curiosity we

have also tried to fit the curve for small and large values of λ/e^2 and arrived at the following result. For small λ/e^2 the function $A(\lambda/e^2)$ seems to start out as

$$A(\lambda/e^2) \approx 125.66(\lambda/e^2)^{1/3} \exp\left(-\frac{4.65}{\lambda/e^2}\right), \quad (73)$$

and the behavior for larger λ/e^2 may be described by

$$A(\lambda/e^2) \approx 100.56 \frac{\exp\left(-1.21(\lambda/e^2)^{2/3}\right)}{(\lambda/e^2)^{1/3}}. \quad (74)$$

However, at the present stage we should not pay too much attention to the precise values of the constants in (73) and (74) since we do not know yet how accurate our approximate calculation of the determinants really is.

Since $m_{U(1)}$ and σ are proportional to $\sqrt{A(\lambda/e^2)}$ the above results show that both quantities vanish in the limits $\lambda/e^2 \rightarrow 0$ and $\lambda/e^2 \rightarrow \infty$. However, physics is very different in these limits. Let us consider the case $\lambda/e^2 \rightarrow \infty$ first. For large values of λ/e^2 the attractive force associated with the Higgs field is clearly short range since the potential term in the classical action (17) forces the Higgs field to unity, i.e., the Higgs field rises from 0 at $r = 0$ to 1 within a distance of the order of $(\lambda/e^2)^{-1}$. Therefore the interaction between the monopoles should indeed be very well described by the magnetic Coulomb force. The result that $m_{U(1)}$ and σ become small as we take λ/e^2 large becomes plausible if we consider the energy of an assembly of monopoles [16]. Classically, this energy is given by the minimum of the three dimensional action (1)

$$S[\hat{A}, \hat{\Phi}] = \int d^3x \left(\frac{1}{4} \hat{F}_{ij}^a \hat{F}_{ij}^a + \frac{1}{2} (\partial_i |\hat{\Phi}|) (\partial_i |\hat{\Phi}|) + \frac{1}{2} |\hat{\Phi}|^2 (\widehat{D}_i \tilde{\Phi})^a (\widehat{D}_i \tilde{\Phi})^a + \frac{\lambda}{8} (|\hat{\Phi}|^2 - v^2)^2 \right). \quad (75)$$

subject to the constraint that the normalized Higgs field $\tilde{\Phi}^a = |\hat{\Phi}|^{-1} \hat{\Phi}^a$ satisfies the required asymptotic properties of magnetic flux and nontrivial homotopy, i.e., [22]

$$\frac{1}{2} \epsilon_{ijk} f^{abc} \partial_i \tilde{\Phi}^a \partial_j \tilde{\Phi}^b \partial_k \tilde{\Phi}^c = 4\pi \sum_n q_n \delta(\mathbf{R} - \mathbf{R}_n). \quad (76)$$

Since according to (76) $\partial_i \tilde{\Phi}$ possesses a $1/r$ singularity at the position of each monopole the quantity $\min_x [(\widehat{D}_i \tilde{\Phi})^a (\widehat{D}_i \tilde{\Phi})^a]$ should increase as the monopole density increases. However, it is easy to see that if we take λ large the potential term

forces $|\hat{\Phi}|^2 \approx v^2$ and it will be energetically favorable for $\min_x[(\widehat{D}_i\tilde{\Phi})^a(\widehat{D}_i\tilde{\Phi})^a]$ to be small. Thus for large values of λ the density of monopoles is small.

If we go to very small values of λ/e^2 the situation is more complicated. First we must check that the superposition principle is still accurate, i.e., the attractive force associated with the Higgs field can be neglected. This is the case if the distance between the monopoles in the gas is large compared to $(\lambda/e^2)^{-1}$. Since the mean distance between the monopoles is proportional to $(e m_{U(1)})^{-2/3}$ (cf. previous section) and $m_{U(1)}$ according to the above results is proportional to $(\lambda/e^2)^{1/3} \exp[-\text{const.}/(\lambda/e^2)]$ the superposition principle indeed seems justified even for very small values of λ/e^2 . To understand the drastic decrease of $m_{U(1)}$ and σ for small values of λ/e^2 let us again consider the classical energy of an assembly of monopoles (75) subject to the constraint (76). We assume the presence of an extremely small monopole density and start to increase it. As the monopole density increases the quantity $\min_x[(\widehat{D}_i\tilde{\Phi})^a(\widehat{D}_i\tilde{\Phi})^a]$ increases. But when this term becomes larger than λv^2 it will be energetically favorable for $|\hat{\Phi}|$ to vanish which now is possible since the potential term in (75) does not constrain $|\hat{\Phi}|$ for small values of λ . But once $|\hat{\Phi}|$ vanishes it will be energetically favorable for the gauge fields to vanish, too.

Indeed a detailed study of the λ/e^2 dependence of the expressions $\alpha_{\mathcal{M}_{\text{FP}}}$, $\beta_{\mathcal{M}_{\text{FP}}}$, $\alpha_{\mathcal{M}}$ and $\beta_{\mathcal{M}}$ shows that in the limit $\lambda/e^2 \rightarrow \infty$ those terms with the highest power in the Higgs self-coupling λ/e^2 become dominant. These are just the terms which result from the Higgs potential. Since they appear with a negative sign in the exponential function (46) $A(\lambda/e^2)$ becomes small as λ/e^2 becomes very large. In the limit $\lambda/e^2 \rightarrow 0$, on the other hand, it is not so that $A(\lambda/e^2)$ vanishes because the Higgs coupling becomes small but because the core of size $1/(\lambda/e^2)$ outside of which the Higgs field approaches its asymptotic form becomes large.

Thus based on the assumption that for small values of the coupling $e^2/4\pi$ the dilute monopole gas partition function is a reasonable approximation to the path integral of the Yang-Mills-Higgs theory the quantum theory undergoes a smooth transition to a non-confining Higgs phase with a massless $U(1)$ gauge boson in the limit $\lambda/e^2 \rightarrow \infty$ and a rapid but still smooth transition to a non-confining

symmetrical phase in the limit $\lambda/e^2 \rightarrow 0$. However, for any finite value of λ/e^2 the quantum theory is confining and all fields massive.

To illustrate both dependencies namely, that on $e^2/4\pi$ and λ/e^2 , the $U(1)$ gauge boson mass is shown in Fig. 4a and Fig. 4b as a function of $e^2/4\pi$ for various values of λ/e^2 . (The corresponding graphs for the string tension look very similar.) Since $m_{U(1)}$ is proportional to $\exp[-m_W C(\lambda/e^2)/2(e^2/4\pi)]$ we see a rapid decrease for very small values of $e^2/4\pi$. But we also see that the curves intersect. The reason for this is that the λ/e^2 -dependence of $m_{U(1)}$ enters in two different ways, namely on the one hand via $A(\lambda/e^2)$ and on the other hand via $\exp[-m_W C(\lambda/e^2)/(e^2/4\pi)]$. Although the λ/e^2 dependence of the mass integral is rather weak as compared to $A(\lambda/e^2)$ it can play a certain role for sufficiently small values of $e^2/4\pi$. To see this let us compare $m_{U(1)}$ for two different values of λ/e^2 , say c_1 and c_2 with $c_1 < c_2 < 2.2$. For $\lambda/e^2 < 2.2$ $A(\lambda/e^2)$ is an increasing function of λ/e^2 whereas $\exp(-m_W C(\lambda/e^2)/(e^2/4\pi))$ is always a decreasing function. Thus if $e^2/4\pi$ becomes sufficiently small $m_{U(1)}(c_1) > m_{U(1)}(c_2)$ but otherwise $m_{U(1)}(c_1) < m_{U(1)}(c_2)$.

6 Summary

The numerical results presented in the previous section show that it is indeed worth studying the dilute monopole gas expansion of the Yang-Mills-Higgs model in more detail, especially, if one views the expansion as an approximation of the Euclidean path integral of this model.

The dilute monopole gas expansion of the Yang-Mills-Higgs model depends on two parameters, $e^2/4\pi$ and λ/e^2 . The $e^2/4\pi$ -dependence becomes already transparent in the formal expression for the monopole partition function whereas the λ/e^2 -dependence can only be seen when both the classical one-monopole mass integral and the corresponding functional determinants have been computed explicitly.

In this paper the determinants have been evaluated with the help of the heat

kernel expansion, an approximation scheme which has been proven successful in a corresponding calculation for instantons. The result of this calculation is an analytical expression which we denoted by $A(\lambda/e^2)$ and which is valid for arbitrary $SU(N)$ gauge groups. However, since apart from the Prasad-Sommerfield limit ($\lambda/e^2 \rightarrow 0$) the monopole solutions are not known analytically the further evaluation of $A(\lambda/e^2)$ has to be done numerically.

As an example we have considered the $SU(2)$ gauge group. Although the mass integral $C(\lambda/e^2)$ is only a rather weakly increasing function between 1.0 and 1.787 its λ/e^2 dependence plays a certain role for small values of the coupling $e^2/4\pi$. The much stronger λ/e^2 dependent quantity is $A(\lambda/e^2)$ which decreases for small and large values of λ/e^2 and has a maximum around $\lambda/e^2 \approx 2.2$. For any finite value of λ/e^2 the dilute monopole gas is confining. However, the string tension and the generated $U(1)$ gauge boson mass are in general very small. In both limits, $\lambda/e^2 \rightarrow \infty$ and $\lambda/e^2 \rightarrow 0$, the system becomes non-confining but for physically different reasons. For $\lambda/e^2 \rightarrow \infty$ we see a smooth transition from the confining phase to the non-confining Higgs phase and for $\lambda/e^2 \rightarrow 0$ a rapid but still smooth transition from the confining phase to the non-confining symmetrical phase.

Although we have seen that the superposition principle seems accurate even for very small values of λ/e^2 one would prefer a direct check. However, this would require to take into account the attractive force associated with the Higgs field which is by far nontrivial. What can certainly be done is to examine the accuracy of the heat kernel approximation used in this paper. To this extent one must compute the next order, i.e., the fourth order in the t -expansion and see whether these higher corrections alter the present results significantly.

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Figure Captions

Figure 1: The monopole mass integral as a function of λ/e^2

Figure 2: The λ/e^2 -dependence which results from the computation of the one-loop corrections to the monopole partition function.

Figure 3: The logarithm of $A(\lambda/e^2)$. A fit for small and large values of λ/e^2 leads to an asymptotic behavior as given in (73) and (74).

Figure 4a: The logarithm of the $U(1)$ gauge boson mass in units of m_w as a function of $e^2/4\pi$ in units of m_W for various values of λ/e^2 ;

---: $\lambda/e^2 = 0.1$, - - - - : $\lambda/e^2 = 0.4$, ———: $\lambda/e^2 = 2$,
 ———: $\lambda/e^2 = 15$, : $\lambda/e^2 = 100$.

Figure 4b: See Fig. 4a.

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